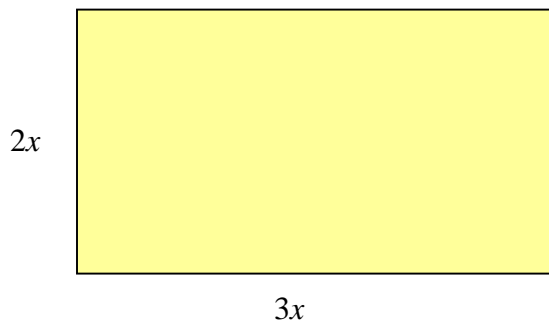


## FOCUS: SOLVING FOR VARIABLES

For some candidates, the presence of variables in a question can cause significant concern. A test-taker with algebra skills will be able to answer some questions more quickly than someone who do not have or recall these concepts. However, there are other ways to determine the correct solution for a multiple-choice question as demonstrated with the sample questions below.

### Example 1:

The dimensions of the rectangle shown below are  $2x$  and  $3x$ .



How many square units are in its area?

- (1) 12
- (2)  $5x$
- (3)  $10x$
- (4)  $5x^2$
- (5)  $6x^2$

Candidates should be familiar with the process of determining the area of a rectangle when the length and width are known. If they have forgotten the formula, it can be found on the Formulas page at the beginning of the *Mathematics Test* booklet. In any case, candidates should recognize that the length and width shown should be multiplied. However, some may have forgotten how to do so when variables are involved.

The key to answering the question without remembering the algebraic process for multiplying variables is to know that the correct response must be true for **any value of  $x$** . Knowing that is true, the following process will lead to the correct solution.

1. Select a value for  $x$ .
2. Evaluate the dimensions of the rectangle using that value.
3. Calculate the area as a number.

- Determine which of the alternatives yields that value for the area when the **same** value is substituted for  $x$ .

**Caution:** When any number can be chosen, avoid selecting 0 or 1. Each of these numbers can lead to a solution that appears to be correct but may not be.

Here is how the process would work in this case.

- Select a value other than 0 or 1 for  $x$ : let  $x = 5$ .
- Evaluate the dimensions of the rectangle:  $2x = 2 \times 5 = 10$ ;  $3x = 3 \times 5 = 15$ .
- Calculate the area:  $10 \times 15 = 150$ .
- Determine which alternative would yield the same area when 5 is substituted for  $x$ :
  - 12 does not change since there is no variable in the expression, and 12 is not the same as 150, the area found in step 3 above.
  - $5x = 5 \times (5) = 25$ ; this is not the same as 150.
  - $10x = 10 \times (5) = 50$ ; this is not the same as 150.
  - $5x^2 = 5 \times (5)^2 = 5 \times 25 = 125$ ; this is not the same as 150.
  - $6x^2 = 6 \times (5)^2 = 6 \times 25 = 150$ ; this is the same area as found before, so this is the correct response.

Urge your students to consider checking their work by selecting another value for  $x$  and observing that the area found using the formula and that found by evaluating the correct alternative (#5) are the same.

- Select 10 for  $x$ .
- $2x = 2 \times 10 = 20$ ;  $3x = 3 \times 10 = 30$
- Area =  $20 \times 30 = 600$
- $6x^2 = 6 \times (10)^2 = 6 \times 100 = 600$ ; this is the same area found in step 3.

This will be true for any number that is selected. If a candidate picked 12.65 for  $x$ , the answers would be the same. However, the arithmetic would be time-consuming without a calculator. The process is easier if your students select simpler numbers, but remind the candidates to avoid picking 0 or 1.

### **Example 2:**

If  $18.5 \times (10 - n) / n = 18.5$ , what is the value of  $n$ ?

- (A) 1
- (B) 2
- (C) 5
- (D) 6
- (E) 10

Again, knowing algebra and number sense would make it easier for a candidate to find the correct solution. However, because this is a multiple-choice question,

the correct response is already among the choices and can be found by substituting the given values for  $n$  and observing what happens.

The two sides of the equation must equal the same number. Since the right-hand side never changes because no variables are present, the left-hand side must be the same number: 18.5. For which of the given alternatives will  $18.5 \times (10 - n) / n$  equal 18.5?

- (A) 1:  $18.5 \times (10 - n) / n = 18.5 \times (10 - 1) / 1 = 18.5 \times (9) / 1 = 18.5 \times 9$ ; without multiplying, candidates should see that this will not equal 18.5
- (B) 2:  $18.5 \times (10 - n) / n = 18.5 \times (10 - 2) / 2 = 18.5 \times (8) / 2 = 18.5 \times 4$ ; without multiplying, candidates should see that this will not equal 18.5
- (C) 5:  $18.5 \times (10 - n) / n = 18.5 \times (10 - 5) / 5 = 18.5 \times (5) / 5 = 18.5 \times 1$ ; candidates should recognize that this equals 18.5

Having found the correct value, there is no need to try the remaining alternatives.

Notice that, in each substitution, both  $n$ 's were replaced by the same number. When substituting a number for a variable, the candidate must use the same number every time the variable appears.

As mentioned about this problem earlier, having number sense would make it easier to find the correct solution. On the left-hand side of the equation, 18.5 is multiplied by an expression containing variables. The result equals the same number because the right-hand side of the equation is 18.5. This can only happen if 18.5 is multiplied by 1. Therefore, the expression  $(10 - n) / n$  must equal 1. Knowing that this is true would make the substitution easier.

- (A) Does  $(10 - 1) / 1 = 1$ ? No,  $(10 - 1) / 1 = (9) / 1 = 9$
- (B) Does  $(10 - 2) / 2 = 1$ ? No,  $(10 - 2) / 2 = (8) / 2 = 4$
- (C) Does  $(10 - 5) / 5 = 1$ ? Yes,  $(5) / 5 = 1$

The process of substituting values for variables is not the most time-efficient way to find the correct answer. However, it is an approach that should be considered if the candidate cannot recall necessary algebra skills. Candidates should consider working on these problems last so that they will have enough time to also work on other questions.